Recent Advances in Image Analysis

Takeshi Yokoo, Andrew Sornborger, Lawrence Sirovich

Laboratory of Applied Mathematics, Department of Biomathematical Sciences, Mount Sinai School of Medicine, New York, NY

Abstract

The mathematical and statistical analysis of high resolution images requires a special consideration. Their large pixel-counts per image make direct pixel-by-pixel calculations impractical, due to the heavy memory and computational demands. We use Principal Component Analysis (PCA) to re-express the dataset in terms of *eigenfunctions*, a collection of basis-images for the most efficient coordinate system to represent the data. PCA provides an ideal platform for subsequent calculations. We have recently developed two new image analysis techniques, *the generalized indicators* and *the method of periodic stacking*, both of which extend PCA by incorporating aspects of signal and noise analyses. Under appropriate data models, they prove to be powerful signal extraction techniques for highly noise-contaminated image data.

Introduction

We consider a population of images, where each image is a two dimensional array of grey-scale pixel values. Since any image of P pixels can be completely specified by P parameters, they can be considered as vectors in P-dimensional space, called the *pixel-space*. This framework allows the use of mathematical approach to the analysis of the image data.

However, there is an inherent problem with representing images in the pixel-space. Even for images of modest size, the pixel-count per image can exceed $O(10^5)$. A linear operator within such a space is a matrix with $O(10^{10})$ entries, which uses gigabytes of memory for storage, and unrealistically long computation time for standard matrix operations. This can severely limit the level of sophistication of the analysis techniques that can be applied to images.

Principal Component Analysis

Sirovich [5] proposed an alternative representation for image data, called the *method of snapshots*, based on a procedure called Principal Component Analysis (a.k.a. SVD, Karhunen-Loeve, intrinsic eigenfunctions, etc.) Instead of specifying images pixel-by-pixel which requires P parameters, this procedure aims to represent images by a linear combinations of the intrinsic features present in the population. By the virtue of the mathematics, PCA generates a set of orthogonal eigenfunctions (images) which can be regarded as basis vectors of the feature space, thus providing an alternative and natural coordinate system in which images can be represented.

Here we have a database of human faces [1], six of which are shown in Fig. 1. Individual faces are representable by superposition of facial features, and some of the basis eigenfunctions are shown in Fig. 2. Typically, there are no more than $O(10^2)$ features in human faces, thus only a few hundred parameters are necessary to specify a face.

PCA thus offers an equivalent but far more efficient framework for image representation than in the original pixel-space. It directly leads to a significant reduction of the computational demands, and provides an ideal working platform for subsequent signal analyses.

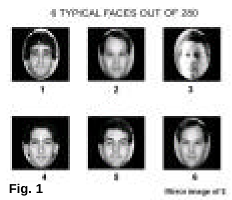

Fig. 1

Fig. 2

Representation of human faces. Fig. 1 shows six instances from a database of faces. To reflect the idea that a mirror image of a face is still a face, we include midline reflections of individual faces in the database. Fig. 2 shows the first six PCA eigenfunctions calculated from the face database. These eigenfunctions represent the cardinal image components (features) intrinsic to the database, with which we can reconstruct any individual face in the dataset.

Generalized Indicator Functions

When data images can be regarded as random vectors, techniques of multivariate analysis are appropriate. The method of *generalized indicator functions* [7] is a new analysis technique derived from discriminant analysis, and it is best suited for the extraction of time-independent signals.

When considering several groups of images, one may be interested in the distinguishing image features among different groups. This can be a challenging task in the presence of large amounts of noise.

For instance, in a functional imaging experiment of visual cortex [6], the images of brain activity are captured while the subject is visually stimulated by drifting grating at various orientations. (Fig. 3)

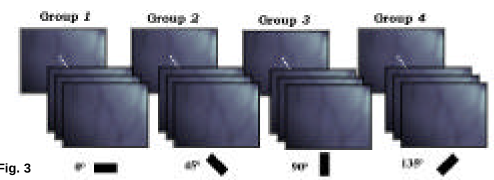

Fig. 3

Optical imaging data of cat visual cortex. Images were acquired while the cat was visually stimulated by drifting grating at 4 different orientations. The orientation specific response cannot be visually detected, as it is approximately $1/10^4$ of the background activity.

The amplitude of the stimulus-specific response (signal) is exceedingly small compared to the ongoing background activity of the cortex (noise) due to blood flow, respiration, spontaneous neural activities and housekeeping metabolism. The signal-to-noise ratio, SNR, can be smaller than $O(10^{-4})$.

One way to treat such noisy data is to average over a large number of images, and compare between groups. (Fig. 4) A more effective approach is to consider a signal as an image feature that is highly correlated with one group, and uncorrelated or anti-correlated with others. This idea can be formalized as a mathematical problem of optimization, which can be solved exactly. The solution images, the *generalized indicators*, are orthogonal images having optimal statistical properties, and form a basis for the signal space in which different groups can be reliably distinguished.

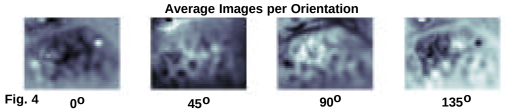

Fig. 4

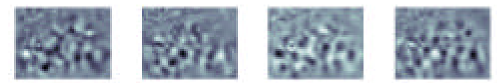

Fig. 5

Cortical response in cat visual cortex to oriented drifting gratings. Fig. 4 shows the average response to 4 orientations, measured from the average response over all orientations. Significant vascular artifacts and residual background activities contaminate the maps of neural response. Fig. 5 shows the reconstructed orientation responses by the generalized indicator method. The background is effectively removed, and the maps of the neural response to oriented stimuli are revealed.

The Method of Periodic Stacking

In signal analysis, the temporal dynamics is often the subject of investigation. The method of periodic stacking [2] is a powerful new procedure for extracting dynamic signals from noisy time-series data.

Consider an experiment which records a noisy response of a system to some input, e.g. an electroencephalogram (EEG) recording in response to some visual stimulation. [4] With multiple recordings of the response to the same stimulus, one can average over the trials to estimate the true response. (red trace, Fig. 6)

The new method, on the other hand, is predicated on the notion that the true response (signal) is reproducible from trial-to-trial. When the response traces from each trial is concatenated back-to-back, the stimulus pattern becomes periodic with respect the concatenated trials, hence the true response is also periodic with the same base frequency.

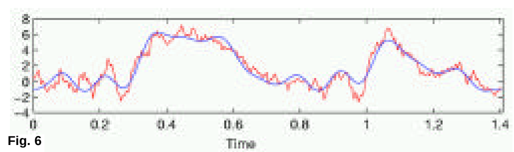

Fig. 6

We use multitaper harmonic analysis, a high-resolution, low bias method for identifying statistically significant harmonic content in the data. As the periodicity of the true response is known, only the appropriate harmonics are extracted, and the rest are considered as noise. This method has proven to be extremely effective in estimating the dynamics of time-dependent signals, e.g. EEG recordings. (blue trace, Fig. 6)

The computational advantage afforded by PCA allows the application of this method to image analysis. Fig. 7 shows images of a mouse heart's electrical activity in re-entrant tachycardia, imaged using voltage sensitive dye [3]. Due to the natural periodicity of tachycardia, this method is an ideal analysis tool for a dataset of this kind. The extracted dynamics (Fig. 8) show a significant noise reduction and a clear wave of action potential sweeping through the ventricular walls.

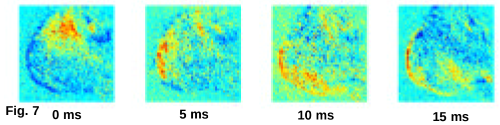

Fig. 7

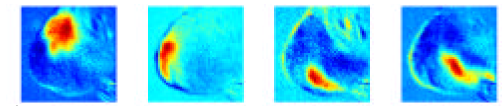

Fig. 8

Arrhythmic electrical activity in mouse heart (re-entrant tachycardia), imaged using voltage sensitive dye. Fig. 7: raw images of the left and the right ventricles, taken at 0, 5, 10, and 15 ms. Fig 8: reconstruction of the time-series by multi-taper harmonic analysis and the periodic stacking method. Significant noise reduction is apparent. The wave of action potentials propagates on the ventricular wall and causes regular cycles of depolarization and repolarization.

Summary

PCA provides a natural and efficient representation of image data, on which sophisticated signal analysis techniques can be applied. We have reviewed two new image analysis techniques developed in this lab, the generalized indicator functions for time-independent signals, and the method of periodic stacking for dynamic signals. We have applied these techniques to a wide variety of noisy biological data, some of which are shown here. They have proved to be highly sensitive and robust procedures for the separation of small signals from noise contaminants. Due to the generality of these procedures, the potential areas of applications may extend beyond that of biomedical data.

Acknowledgements

We would like to thank those who generously shared their data with us: A.K. Prashanth for the optical imaging data, Greg Morley for the mouse heart data, and Arnaud Delorne for the EEG data. This research is supported by Defense Advanced Research Projects Agency (DARPA) MDA 972-01-1-0028.

References

[1] Kirby, M. and Sirovich, L. 1990 Application of the Karhunen-Loeve procedure for the characterization of human faces. *IEEE Trans. Pattern Analysis and Machine Intelligence*. **12**

[2] Sornborger, A., Sailstad, C., Kaplan, E., Sirovich L. Spatio-Temporal Analysis of Optical Imaging Data (submitted to *NeuroImage*)

[3] Sornborger, A., Morley, G., Sirovich L. A Method for Denoising Periodic Multivariate Signals: Application to Voltage Sensitive Dye Imaging Data of the Mouse Heart. (in preparation)

[4] Sornborger, A., Delorne, A., Sailstad, C., Sirovich L. Detection and Extraction of Average and Differential Dynamical Responses to Multiple Stimuli. (in preparation)

[5] Sirovich, L. and Everson, R.E. 1992 Analysis and management of large scientific databases. *Int. J. Supercomput. Appl.* **6**:50-68

[6] Sirovich, L. and Kaplan, E. 2001 Analysis methods for optical imaging. In *Methods for in Vivo Optical Imaging of the Central Nervous System* (R. Frostig, Ed.) CRC Press, Boca Raton, FL, in press.

[7] Yokoo, T., Knight, B.W., Sirovich, L. 2001 Optimization Approach to Signal Extraction from Noisy Multivariate Data (to appear in *NeuroImage*)